1.3. VARIABLE SEPARABLE DIFFERENTIAL EQUATION

1.3.1. Introduction

Separable differential equations are a special type of differential equation. The variables involved in this equation can be separated to get the solution to the equation.

It is represented as:

$$\frac{dy}{dx} = f(x)\, g(y) \quad \text{....(1)}$$

where x and y are the variables and they are clearly separated from each other. Once variables get separated then by integrating both sides of the equation we can easily determine the solution of the differential equation.

After the separation of variables the equation (1) is written as: $\frac{d(y)}{g(y)} = f(x)\, dx$

We have the first order and first degree differential equation as:

$$f_1(x)g_2(y)dx + f_2(x)g_1(y)dy = 0 \quad \text{...(2)}$$

Then equation (2) can also be written as;

$$M(x)dx + N(y)dy = 0 \quad \text{.....(3)}$$

where $M(x) = (f_1/f_2)(x)$, is a pure function of x;

$N(y) = (g_1/g_2)(y)$ is a pure function of y

Therefore the equation can be easily integrated and we can get the solution.

Note: If in an equation, it is possible to get all the functions of x and dx to one side and all the functions of y and dy to the other, the variables are said to be separable.

1.3.2. Process of Solving an Equation in which Variable are Separable

To understand the processes of solving separable differential equation follow these basic steps:

Step 1: Write the derivative as a product of functions of individual variables,

i.e., $\frac{dy}{dx} = f(x)\, g(y)$.

Step 2: Separate the variables by writing them on each side of the equation ,

i.e., $\frac{d(y)}{g(y)} = f(x)\, dx$.

Step 3: Integrate both sides and find the value of y, and hence the general solution of the separable differential equation,

i.e., $\int \frac{dy}{g(y)} = \int f(x)\, dx$

1.3.3. Different Types of Variable Separable Differential Equation

Following are the different cases of Different Types of Variable Separable Differential Equation:

1) **Differential Equation of the Type $\frac{dy}{dx} = f(x)$**

We have $\frac{dy}{dx} = f(x)$ ....(1)

Then $dy = f(x)\, dx$ ...(2)

Integrating Equation (2) on both sides, we get

$\int dy = \int f(x)\, dx$

$\Rightarrow\ y = \int f(x) dx + c$

where c is constant and it is the solution.

Example 34: Find the solution of $(e^x + e^{-x})\frac{dy}{dx} = (e^x - e^{-x})$

Solution: We have

$\Rightarrow (e^x + e^{-x})\frac{dy}{dx} = (e^x - e^{-x})$ ...(1)

$\Rightarrow dy = \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$ ...(2)

Integrating both sides of equation (2), we obtain

$\Rightarrow \int dy = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx \quad \Rightarrow \int dy = \int \frac{1}{t} dt$,

Where $t = e^x + e^{-x}$

$\Rightarrow\ y = \log|t| + c$

$\Rightarrow\ y = \log|e^x + e^{-x}| + c$ ...(3)

Equation (3) is the required solution of equation (1).